MULTIPERIODIC SOLUTION OF A BOUNDARY VALUE TASK FOR THE SYSTEM OF PARABOLIC EQUATIONS

МУЛЬТИПЕРИОДИЧЕСКОЕ РЕШЕНИЕ КРАЕВОЙ ЗАДАЧИ ДЛЯ СИСТЕМЫ ПАРАБОЛИЧЕСКИХ УРАВНЕНИЙ

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We consider the boundary value problem

$$Lu \equiv \frac{\partial u}{\partial \tau} + \sum_{j=1}^{m} \frac{\partial u}{\partial t_j} - \Delta u - \frac{\partial^2 u}{\partial y^2} + \gamma u = f(\tau, t, x, y), \tag{1}$$

$$u(\tau, t, x, 0) = \Psi(\tau, t, x), \tag{2}$$

where multivariate time $(\tau, t) \in E_{1+m}$, $x \in E_n$, E_n - n- measured of the Euclid space, $y \in E_1^+ = [0, +\infty)$, $(\tau, t, x, y) \in E^+_{1+m+n+1}$; $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + ... + \frac{\partial^2}{\partial x_n^2}$- operator of Laplace; $\gamma = const > 0$; $f(\tau, t, x, y)$ and $\Psi(\tau, t, x)$ - n – vector – functions.

For functions $f(\tau, t, x, y)$ and $\Psi(\tau, t, x)$ are expected that meet the conditions of periodicity:

$f(\tau + \theta, t + k\omega, x + p\sigma, y) = f(\tau, t, x, y)$, $k \in Z^m$, $p \in Z^n$,

$\Psi(\tau + \theta, t + k\omega, x + p\sigma) = \Psi(\tau, t, x)$, $k \in Z^m$, $p \in Z^n$,

where $\theta, \omega_1, ..., \omega_m, \sigma_1, ..., \sigma_n$ -periods, $k\omega = (k_1\omega_1, k_2\omega_2, ..., k_m\omega_m) -$ m-vector, $p\sigma = (p_1\sigma_1, p_2\sigma_2, ..., p_n\sigma_n)$- n -vector;

condition of Gelder on τ, t, x, y with exponents $\frac{\alpha}{2}, \alpha$:

$$|f(\bar{\tau}, \bar{t}, \bar{x}, \bar{y}) - f(\tau, t, x, y)| < \Gamma_1 \left(|\bar{\tau} - \tau|^{\frac{\alpha}{2}} + |\bar{t} - t|^{\alpha} + |\bar{x} - x|^{\alpha} + |\bar{y} - y|^{\alpha} \right),$$

$$|\Psi(\bar{\tau}, \bar{t}, \bar{x}) - \Psi(\tau, t, x)| < \Gamma_2 \left(|\bar{\tau} - \tau|^{\frac{\alpha}{2}} + |\bar{t} - t|^{\alpha} + |\bar{x} - x|^{\alpha} \right),$$

where $\alpha \in (0,1)$, $\Gamma_1, \Gamma_2 - const$.

Problem 1. Find sufficient conditions existence and uniqueness (θ, ω, σ) - periodic on τ, t, x of solutions of the equations (1) with condition (2).

For research the problem 1 consider auxiliary problem.

Problem 2. Find (θ, ω, σ) - periodic on τ, t, x of solutions of the equations

$$L\bar{u} = \bar{f}(\tau, t, x, y), \tag{3}$$

with conditions

$$\bar{u}(\tau_0, t, x, y) = \bar{\varphi}(t, x, y) \in CB(E_{m+n+1}), \tag{4}$$

$$\bar{u}(\tau, t, x, 0) = \Psi(\tau, t, x), \tag{5}$$

where $\bar{f}(\tau, t, x, y) = \begin{cases} f(\tau, t, x, y), & for\ y \geq 0 \\ -f(\tau, t, x, -y), & for\ y < 0 \end{cases}$;

$$\overline{\varphi}(t,x,y)=\begin{cases}\varphi(t,x,y), & for\ y\geq 0\\ -\varphi(t,x,-y), & for\ y<0\end{cases},$$

$CB(E_{m+n+1})$ - Banach space continuous function $\overline{\varphi}(t,x,y)$ with norm

$\|\overline{\varphi}\|_{CB(E_{m+n+1})}=\sup\limits_{E_{m+n+1}}|\overline{\varphi}(t,x,y)|$.

In this abstract using idea works [1]-[3].

Fundamental of solution differential equations of parabolic type were researched in works many authors, note the work [4].

The solution of problem (3)-(5) search in the shape

$$\overline{u}(\tau,t,x,y)=\ell^{-\gamma(\tau-\tau_0)}\int\limits_{E_n}V(\tau-\tau_0,x-\xi)\times$$
$$\times\int\limits_{-\infty}^{+\infty}U(\tau-\tau_0,y-\eta)U_0(\tau-\tau_0,t-e\tau+e\tau_0)\overline{\varphi}(t-e\tau+e\tau_0,\xi,\eta)d\eta d\xi+$$
$$+\int\limits_{\tau_0}^{\tau}\ell^{-\gamma(\tau-s)}\int\limits_{E_n}V(\tau-s,x-\xi)\int\limits_{-\infty}^{+\infty}U(\tau-s,y-\eta)\times$$
$$\times U_0(\tau-s,t-e\tau+es)\bar{f}(s,t-e\tau+es,\xi,\eta)d\eta d\xi ds+$$
$$+\int\limits_{\tau_0}^{\tau}\ell^{-\gamma(\tau-s)}\int\limits_{E_n}V(\tau-s,x-\xi)\frac{\partial U}{\partial\eta}(\tau-s,y-\eta)\times$$
$$\times U_0(\tau-s,t-e\tau+es)\Psi(s,t-e\tau+es,\xi)d\xi ds. \qquad (6)$$

Here $U(\tau-\tau_0,y-\eta)=[4\pi(\tau-\tau_0)]^{-\frac{1}{2}}\ell^{-\frac{|y-\eta|^2}{4(\tau-\tau_0)}}$ for $\tau>\tau_0$ fundamental of solution of the equation $\frac{\partial u}{\partial\tau}-\frac{\partial^2 u}{\partial y^2}=0$, function of $V(\tau-\tau_0,x-\xi)=[4\pi(\tau-\tau_0)]^{-\frac{n}{2}}\ell^{-\frac{|x-\xi|^2}{4(\tau-\tau_0)}}$ - fundamental of solution of the equation $\frac{\partial u}{\partial\tau}-\frac{\partial^2 u}{\partial x^2}=0$ for $\tau>\tau_0$, $V(\tau-\tau_0,x-\xi)U(\tau-\tau_0,y-\eta)U_0(\tau-\tau_0,t-e\tau+e\tau_0)\ell^{-\gamma(\tau-\tau_0)}$ - fundamental of solution the operator of L for all $x,\xi\in E_n$, $\tau>\tau_0$.

For $\tau<\tau_0$ fundamentals of solution continue zero.

Use the condition of periodically:

$\overline{u}(\tau_0,t,x,y)=\overline{u}(\tau_0+\theta,t,x,y)$

search among solutions (6) θ - periodic the solution of problem.

Then

$$u^*(\tau,t,x,y)=\int\limits_{-\infty}^{\tau}\ell^{-\gamma(\tau-s)}\int\limits_{E_n}V(\tau-s,x-\xi)U_0(\tau-s,t-e\tau+es)\times$$
$$\times\int\limits_{0}^{+\infty}[U(\tau-s,y-\eta)-U(\tau-s,y+\eta)]f(s,t-e\tau+es,\xi,\eta)d\eta d\xi ds+$$
$$+\int\limits_{-\infty}^{\tau}\ell^{-\gamma(\tau-s)}\int\limits_{E_n}V(\tau-s,x-\xi)\frac{\partial U}{\partial\eta}(\tau-s,y-\eta)\Big|_{\eta=0}\times$$
$$\times U_0(\tau-s,t-e\tau+es)\Psi(s,t-e\tau+es,\xi)d\xi ds. \qquad (7)$$

Some quality of the function of $u^*(\tau,t,x,y)$:

1. $|u^*(\tau,t,x,y)|\leq\frac{M_0M_1}{\gamma}+N\ell^{-\sqrt{\gamma}y}$,

where $N = M_0 M - \frac{M_0 M_1}{\gamma}$ - $const$.

2. The function of $u^*(\tau,t,x,y)$ multiperiodic on τ,t,x:

$u^*(\tau+\theta, t+k\omega, x+p\sigma, y) = u^*(\tau,t,x,y)$.

3. The function of $u^*(\tau,t,x,y)$ uniqueness.

Theorem. If for functions $f(\tau,t,x,y)$ and $\Psi(\tau,t,x)$ were executed conditions, then equation (1) by condition (2) and $\gamma = const > 0$ has uniqueness multiperiodic on τ, t, x of solutions $u^*(\tau,t,x,y)$, present in the shape (7).

REFERENCES LIST

1. *Umbetzhanov D.U.* Almost periodic solutions of evolutions equations. Alma-Ata: Nayka, 1990. 184p. (in Russian).
2. *Sartabanov Zh.A.* Condition of periodically of differential systems solution with multidimensional time // Izvestiya NAN RK. Ser. fiz.- mat. 2004. No.3, pp.44-48. (in Russian).
3. *Asanova A.T.* Limited solution of the nonlinear parabolic equations // Izvestiya MN-AN RK. Ser. fiz.- mat. 1997. No.1, pp.33-39 (in Russian).
4. *Friedman A.* Partial differential equations of parabolic type. M.: Mir, 1968. 428p. (inRussian).

ОБ УСЛОВИЯХ РАЗРЕШИМОСТИ НЕЛОКАЛЬНОЙ КРАЕВОЙ ЗАДАЧИ ДЛЯ СИСТЕМЫ УРАВНЕНИЯ В ЧАСТНЫХ ПРОИЗВОДНЫХ

ON CONDITIONS FOR A NONLOCAL BOUNDARY VALUE SYSTEM EQUATIONS IN PARTIAL DERIVATIVE

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На $\overline{\Omega} = \{(x,t): at \le x \le at+\omega, 0 \le t \le T\}, T > 0, \omega > 0$ рассматривается краевая задача с нелокальным условием для системы уравнений в частных производных

$$D\left[\frac{\partial}{\partial x}u\right] = A(x,t)\frac{\partial u}{\partial x} + S(x,t)u + f(x,t), \quad u \in R^n, \tag{1}$$

$$B(x)\frac{\partial u}{\partial x}(x,0) + C(x)\frac{\partial u}{\partial x}(x+T,T) = d(x), \quad x \in [0,\omega], \tag{2}$$

$$u(t,t) = \Psi(t), \qquad t \in [0,T], \tag{3}$$

где $D = \frac{\partial}{\partial t} + a\frac{\partial}{\partial x}$, $a = (a_1, a_2, ..., a_n)$ - n - вектор; $A(x,t)$, $S(x,t)$ – $(n \times n)$-матрицы, $f(x,t)$ - n - вектор-функция непрерывны по x и t на $\overline{\Omega}$; $B(x), C(x)$ – $(n \times n)$ – матрицы и n - вектор-функция $d(x)$ – непрерывны на $[0,\omega]$; функция $\Psi(t)$ непрерывно дифференцируема на $[0,T]$.

Через $C(\overline{\Omega}, R^n)$ обозначим пространство непрерывных по x и t функций $u: \overline{\Omega} \to R^n$ с нормой $\|u\|_0 = \max_{x\in[0,\omega]}\max_{t\in[0,T]}\|u(x,t)\|$; $\|A\|_0 = \max_{(x,t)\in\Omega}\|A(x,t)\| = \max_{(x,t)\in\Omega}\max_{i=1,n}\sum_{j=1}^{n}|a_{ij}(x,t)|$,

$\|d\|_1 = \max_{x\in[0,\omega]}\|d(x)\|$, $\|\Psi\|_2 = \max_{t\in[0,T]}\|\Psi(t)\|$, .

Цель данного сообщения - найти коэффициентные достаточные условия корректной разрешимости задачи (1)-(3).

На современном этапе развития теории краевых задач для уравнения в частных производных значительный интерес представляют задачи с нелокальными ограничениями, в